



1. The hyperbola  $H$  has foci at  $(5, 0)$  and  $(-5, 0)$  and directrices with equations

$$x = \frac{9}{5} \text{ and } x = -\frac{9}{5}.$$

Find a cartesian equation for  $H$ .

(7)

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3. The point  $P$  lies on the ellipse  $E$  with equation

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

$N$  is the foot of the perpendicular from point  $P$  to the line  $x = 8$

$M$  is the midpoint of  $PN$ .

- (a) Sketch the graph of the ellipse  $E$ , showing also the line  $x = 8$  and a possible position for the line  $PN$ . (1)
  
- (b) Find an equation of the locus of  $M$  as  $P$  moves around the ellipse. (4)
  
- (c) Show that this locus is a circle and state its centre and radius. (3)

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4. The plane  $\Pi_1$  has vector equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix} + s \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} + t \begin{pmatrix} 1 \\ 2 \\ -2 \end{pmatrix},$$

where  $s$  and  $t$  are real parameters.

The plane  $\Pi_1$  is transformed to the plane  $\Pi_2$  by the transformation represented by the matrix  $\mathbf{T}$ , where

$$\mathbf{T} = \begin{pmatrix} 2 & 0 & 3 \\ 0 & 2 & -1 \\ 0 & 1 & 2 \end{pmatrix}$$

Find an equation of the plane  $\Pi_2$  in the form  $\mathbf{r} \cdot \mathbf{n} = p$

(9)

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6. It is given that  $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$  is an eigenvector of the matrix **A**, where

$$\mathbf{A} = \begin{pmatrix} 4 & 2 & 3 \\ 2 & b & 0 \\ a & 1 & 8 \end{pmatrix}$$

and  $a$  and  $b$  are constants.

(a) Find the eigenvalue of **A** corresponding to the eigenvector  $\begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$ . **(3)**

(b) Find the values of  $a$  and  $b$ . **(3)**

(c) Find the other eigenvalues of **A**. **(5)**

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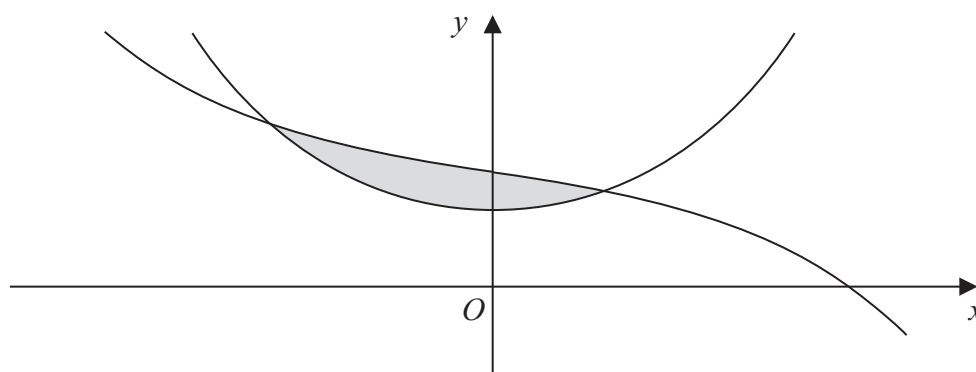
Question 6 continued

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7.



**Figure 1**

The curves shown in Figure 1 have equations

$$y = 6 \cosh x \text{ and } y = 9 - 2 \sinh x$$

- (a) Using the definitions of  $\sinh x$  and  $\cosh x$  in terms of  $e^x$ , find exact values for the  $x$ -coordinates of the two points where the curves intersect.

**(6)**

The finite region between the two curves is shown shaded in Figure 1.

- (b) Using calculus, find the area of the shaded region, giving your answer in the form  $a \ln b + c$ , where  $a$ ,  $b$  and  $c$  are integers.

**(6)**

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8.

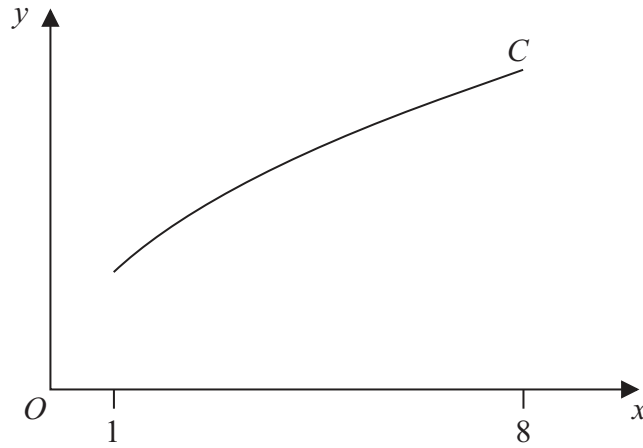


Figure 2

The curve  $C$ , shown in Figure 2, has equation

$$y = 2x^{\frac{1}{2}}, \quad 1 \leq x \leq 8$$

(a) Show that the length  $s$  of curve  $C$  is given by the equation

$$s = \int_1^8 \sqrt{\left(1 + \frac{1}{x}\right)} dx \tag{2}$$

(b) Using the substitution  $x = \sinh^2 u$ , or otherwise, find an exact value for  $s$ .

Give your answer in the form  $a\sqrt{2} + \ln(b + c\sqrt{2})$  where  $a$ ,  $b$  and  $c$  are integers. (9)

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**Question 8 continued**

Handwriting lines for the answer to Question 8.

**Q8**

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**(Total 11 marks)**

**TOTAL FOR PAPER: 75 MARKS**

**END**

